On the Suggestion made by Sir David Gill that the Brighter Fixed Stars are as a whole rotating with respect to Fainter Stars as a whole. *By* H. H. Turner, *D.Sc.*, *F.R.S.*, *Savilian Professor.*

1. In No. 3800 of the *Astronomische Nachrichten* there is a "Preliminary Note on an Apparent Rotation of the Brighter Fixed Stars as a whole with respect to Fainter Stars as a whole," by Sir David Gill. He found on comparing the Cape Catalogues for 1880 and 1900 that the "apparent systematic corrections . . . could not be reconciled by any of the usual hypotheses, such as parallactic motion due to translation of the Sun in space, or systematic error in R.A. depending upon magnitude." He found similar difficulties on proceeding to compare these catalogues with Taylor's Madras (1835) Catalogue, as re-reduced by Downing, and by comparisons with the Cordoba General Catalogue and Gould's Zones; and considers that we are "driven to the conclusion that the brighter stars rotate, with respect to the fainter stars as a whole, about some centre, or that Newcomb's proper motions and precession do not give star-places which would approximately represent fundamental determinations of the first point of *Aries*, as defined with respect to a much larger number of stars. Even this latter hypothesis would not account for all the facts observed."

2. Such important conclusions are deserving of the earliest possible scrutiny by any independent method likely to throw any light upon them; and as we have at Oxford material for such a scrutiny, in the shape of measures of photographs extending over nearly ten years, no time was lost in examining these measures for the phenomenon suggested by Sir David Gill. I gratefully acknowledge the help of Mr. F. C. H. Carpenter, of the Durham Observatory, in the preliminary part of this work, on which he spent many hours during a holiday visit to Oxford.

3. The question to be decided, having regard to the material available, is briefly this: Does the "magnitude-equation" of the Cambridge meridian observations, as deduced from our photographic measures, remain constant, whatever be the date when the plates are taken, or does it change with the date?* A determination of this magnitude equation from photographs taken during the years 1892–9 was given in *Monthly Notices*, vol. lx. p. 3, the

* It is, of course, a distinct disadvantage to have to bring in the Cambridge magnitude equation at all. A much more direct method of attacking the problem would be to take plates of the same regions at different epochs and compare the positions of the stars upon them. But at Oxford we have had to be content, so far, with photographing each region once only; and the Cambridge observations must be used as a connecting link.

assumption being made that the proper motions of stars of any given magnitude are in the mean zero; and that the difference of epoch between the Cambridge meridian observations (about 1880·0) and that of the plates (about 1896·0) had therefore no effect on the result. It is just this assumption which we are now to test. Given a sufficient lapse of time, there is no difficulty in testing it thoroughly, but it was somewhat doubtful whether a sufficient interval was covered by the plates taken at Oxford. They run from 1892 to the present time, about ten years; but as a considerable number of both early and late plates are required to make the comparison effective, and as the plates must also be distributed with some approach to uniformity round the 24^h of R.A., the maximum interval is reduced in practice to four or five years.

4. As a preliminary experiment the residuals Oxford—Cambridge in R.A. (or rather in x-coordinate) were formed for all photographs taken earlier than 1894·0. These were compared with those found in the paper of 1899 November, which referred to a later epoch on the average; and it was found that the deduced magnitude-equation was more pronounced for the earlier results—i.e. *the R.A.'s of bright stars (mags.* 5·0 *to* 8·0) *are decreasing relatively to those of faint stars (mags.* 8·0 *to* 9·5) *in the belt* $+25°$ *to* $+30°$ *Decl.*

This conclusion, which was scarcely more than suggested by the preliminary work, was confirmed by a more extended inquiry; and it does not seem necessary to give the earlier work, which needs several qualifications. But the conclusion is stated at once to draw attention to the unexpected result that the motion of the bright stars is in the *opposite direction* to that found by Sir David Gill. In his belt of the heavens ($-40°$ to $-52°$) the R.A.'s of the bright stars are *increasing*; and we may also remark here the amount of the increase, which is

$$+0^s{\cdot}0015 \text{ per magnitude per year}$$

as found by a comparison of Cape (1880·0) and Cape (1900·0);

$$+0^s{\cdot}0013 \text{ per magnitude per year}$$

as found by a comparison of Taylor (1835·0) and Cape (1900·0).

It will be seen in what follows that the quantities found at Oxford are very like these in amount, but of the contrary sign.

5. The preliminary inquiries having shown that a more detailed investigation might be profitably undertaken, the dates of exposure of all the plates which had been measured were examined, and it was found that in three zones of plates, $+26°$, $+27°$, and $+29°$, groups could be formed in each octant of R.A. of both "early" and "late" plates, the interval being something like five years on the average. The mean dates of the groups are given in Table I., subtracting 1890·0 in each case.

TABLE I.

Mean Date (subtracting 1890·0) *and No. of Plates in each Group.*

	Zone + 26°.					Zone + 27°.					Zone + 29°.				
	Early.		Late.			Early.		Late.			Early.		Late.		
h h	Mean Date.	No. Plates.	Mean Date.	No. Plates.	Inter-val. y	Mean Date.	No. Plates.	Mean Date.	No. Plates.	Inter-val. y	Mean Date.	No. Plates.	Mean Date.	No. Plates.	Inter-val. y
0·0– 3·0	3·6	4	8·0	5	4·4	(4·7)*	0	9·4	2	(4·7)*	5·9	8	11·8	9	5·9
3·0– 6·0	3·6	5	9·6	8	6·0	5·0	13	9·1	9	4·1	6·1	5	11·4	10	5·3
6·0– 9·0	3·2	5	9·5	9	6·3	5·1	15	9·3	7	4·2	6·0	9	11·2	8	5·2
9·0–12·0	3·2	5	9·2	9	6·0	5·1	1	9·6	16	4·5	6·1	7	11·6	13	5·5
12·0–15·0	3·6	7	10·1	11	6·5	5·3	4	9·6	10	4·3	6·3	18	11·4	2	5·1
15·0–18·0	5·4	2	9·6	12	4·2	5·4	5	9·6	15	4·2	5·9	12	11·5	8	5·6
18·0–21·0	3·3	12	7·7	12	4·4	3·5	6	7·8	6	4·3	5·2	15	10·3	5	5·1
21·0–24·0	3·6	12	7·8	1	4·2	4·4	8	8·2	9	3·8	5·0	18	9·3	2	4·3
Sum or Mean		52		67	5·2		52		74	4·3		92		57	5·2

* The mean date for zones + 26° and + 29° has been substituted.

It will be seen that some of the groups contain only a small number of plates, which is natural, seeing that the work was arranged without any reference to the present purpose. But it will be presently shown that all three zones may be combined, and then this weakness disappears.

6. The measures of these plates were utilised in the following way: from the Cambridge Catalogue (series of the *Astr. Gesellsch.* + 25° to + 30°, epoch 1875·0) the R.A.'s and declinations have been brought up to 1900·0 and converted into standard coordinates: these are used to determine the plate constants, *i.e.* the coefficients in linear expressions

$$\xi - x = ax + by + c$$
$$\eta - y = dx + ey + f$$

and using these plate constants to correct the measures, residuals are formed (Oxford — Cambridge) for the individual stars. All this is the ordinary course of the work for the Astrographic Catalogue. For the present purpose these residuals were collected into the following groups, according to the star magnitude as given in the Cambridge Catalogue.

TABLE II.

Limits of Magnitude.	Mean Mag.	Total No. Stars.					
		+ 26°.		+ 27°.		+ 29°.	
		E.	L.	E.	L.	E.	L.
5·0 to 6·9	6·0	60	62	62	80	89	43
7·0 „ 7·9	7·5	151	201	167	189	274	145
8·0 „ 8·8	8·5	546	649	544	651	951	486
8·9 „ 9·1	9·0	543	682	615	678	1126	562
9·2 „ 9·5	9·4	683	801	599	793	1335	534

The "mean mag." for the group was adopted as a standard for reference, and if the mean magnitude for any group came out (say) 6·25, a correction was applied to reduce the result to 6·0, using for the small interval 0·25 our general knowledge of the Cambridge magnitude equation in a manner which needs no explanation. No stars of magnitude brighter than 5·0 were included, for reasons which will be tolerably obvious on inspection of the diagram given in *Monthly Notices*, lx. p. 5.

The totals shown in Table II. will give a general idea of the amount of material available.

7. We may now set down the means of these groups, which form the raw material on which the present discussion is based. The unit of the following tables is 0·0001 of a réseau interval, or $0''\cdot03$, or $0^{s}\cdot00222$ at 26°, $0^{s}\cdot00224$ at 27°, and $0^{s}\cdot00229$ at 29°; we may take $0^{s}\cdot00225$ as the mean value in R.A. The letters E and L are used to indicate the "early" and "late" plates of Table I.

TABLE III.

Mean x Residual (Oxford – Cambridge) for each group in units of $0''\cdot03$ *or* $0^{s}\cdot00225$.

Zone +26°.

Octant of R.A.	Mag. 6·0		Mag. 7·5.		Mag. 8·5.		Mag. 9·0.		Mag. 9·4.	
h h	E.	L.	E.	L.	E.	L.	E.	L.	E.	L.
0– 3	+56	+ 4	+85	+34	+18	+21	− 4	−12	−36	−28
3– 6	+37	+46	− 2	+23	+ 5	+24	− 2	− 2	−24	−41
6– 9	+64	+46	+38	+46	+18	+17	− 2	+ 2	−41	−28
9–12	+29	+43	+34	+19	+18	+10	− 8	− 3	−29	−25
12–15	+34	+36	+35	+32	+16	+16	+ 8	− 9	−43	−40
15–18	+90	+71	+44	+37	+14	+14	−10	+ 2	−26	−31
18–21	+44	+56	+34	+31	+24	+16	+ 3	+ 5	−28	−28
21–24	+61	−36	+41	+17	+21	+28	+ 7	+ 7	−32	−17
Mean	+51·9	+33·3	+38·6	+29·9	+16·8	+18·3	− 1·0	− 1·3	−32·4	−29·8

Zone +27°.

Octant of R.A.	Mag. 6·0		Mag. 7·5.		Mag. 8·5.		Mag. 9·0.		Mag. 9·4.	
h h	E.	L.	E.	L.	E.	L.	E.	L.	E.	L.
0– 3	(+70)	+89	(+71)	+72	(+24)	+ 8	(−3)	− 9	(−33)	−38
3– 6	+35	+35	+23	+34	+14	+29	−5	− 3	−35	−43
6– 9	+39	+27	+40	+15	+15	+21	−4	+ 2	−34	−24
9–12	+94	+39	(+26)	+30	+33	+19	−3	−12	−39	−24
12–15	+44	+32	+41	+17	+13	+13	−8	− 4	−54	−32
15–18	+49	+49	+46	+25	+ 6	+11	− 2	+ 4	−25	−35
18–21	+48	+32	+40	+33	+19	+16	− 1	+ 3	−26	−27
21–24	+37	+58	+35	+39	+21	+29	+2	+ 2	−27	−30
Mean	+52·0	+45·1	+40·3	+33·1	+18·1	+18·3	−3·0	− 2·1	−34·1	−31·6

Zone +29°.

Octant of R.A.	Mag. 6·0. E.	L.	Mag. 7·5. E.	L.	Mag. 8·5. E.	L.	Mag. 9·0. E.	L.	Mag. 9·4. E.	L.
h h										
0– 3	+84	+53	+57	+32	+30	+13	− 2	−20	−29	−23
3– 6	+38	+43	+29	+11	+ 6	+ 9	−10	− 5	−38	−22
6– 9	+36	+72	+37	+15	+12	+19	0	+ 2	−32	−23
9–12	+46	+16	+18	+41	+15	+14	−12	− 2	−18	−27
12–15	+36	+ 5	+35	+35	+17	+14	− 5	− 7	−39	−38
15–18	+35	+26	+15	+24	+12	+14	− 4	+ 2	−23	−29
18–21	+49	+18	+35	+34	+21	+13	0	+ 3	−24	−25
21–24	+65	+35	+41	+37	+28	+17	+ 3	−12	−29	−26
Mean	+48·6	+33·5	+33·4	+28·6	+17·6	+14·1	− 3·8	− 4·9	−29·0	−26·6

8. Now in studying such phenomena as have been suggested by Sir D. Gill he insists that "we must give equal weight to groups of stars which are symmetrically distributed in right ascension." To take the simple means of the eight octants in Table III. and compare them does not fulfil this condition; for the intervals (as may be seen by referring to Table I.) are not the same in the different octants, and an octant with a long interval will thus have virtually greater weight. But before proceeding to make this correction we may glance at the *sort* of result we may expect, which will not be very different from that obtained from these simple means, since the intervals are roughly about five years.

Table IV.

Approximate Mean P.M.'s in R.A. for five years in units of 0″·03 *or* 0ˢ·00225.

Decl.	6·0.	7·5.	8·5.	9·0.	9·4.
+26°	−18·6	−8·7	+1·5	−0·3	+2·6
+27	− 6·9	−7·2	+0·2	+0·9	+2·5
+29	−15·1	−4·8	−3·5	−1·1	+2·4
Mean	−13·5	−6·9	−0·6	−0·2	+2·5

9. These numbers are sufficient to indicate:

(*a*) The *real* nature of the phenomenon from the fact that there is a sensible accordance between all these zones, which are quite independent.

(*b*) The *sign* of the motion is a *decreasing* R.A. for bright stars relatively too faint.

(*c*) The magnitude of the motion is about 1 unit per magnitude per year, which would give a series of theoretical results, such as

Mag.		6·0.	7·5.	8·5.	9·0.	9·4.
Calculated	...	−15·0	−7·5	−2·5	0·0	+2·0
Observed	...	−13·5	−6·9	−0·6	−0·2	+2·5
O.−C.	...	+ 1·5	+0·6	+1·9	−0·2	+0·5

(The residuals would be improved a little by assuming a different rate for bright and faint stars; but for the present we will put this aside.) This quantity $0''.03$ or $0^s.00225$ per magnitude per year is nearly double the quantity found by Sir D. Gill (see § 4); but it is at any rate of the same order of magnitude.

10. With these indications of the reality of the phenomenon we may proceed to combine our results to the best advantage. There seems no need to keep the results of the different zones separate, and by combining them we shall remove some of the weaknesses made apparent in Table I. The number of plates in each octant will then stand as follows:

Table V.

	0^h-3^h	-6^h	-9^h	-12^h	-15^h	-18^h	-21^h	-24^h
Num. early plates	12	23	29	13	29	19	33	38
,, late ,,	16	27	24	38	23	35	23	12
Mean interval in years	5·2	5·2	4·9	5·3	4·5	4·6	4·1	4·0

which should give a strong enough determination of the quantity throughout the R.A. circuit.

11. In combining the results for different zones weights have been assigned according to the number of *plates*, so as to have a uniform system of weighting for stars of different magnitudes and for the mean dates. The results are given in Table VI. The differences between early and late plates have then been divided by the mean intervals shown in Table V. so as to get the annual P.M.'s shown in Table VII., still expressed in the unit $0''.03$ or $0^s.00225$.

Table VII.

Deduced Relative Motions in each Octant.

(The unit is $0''.03$ or $0^s.00225$ per year.)

Octant.	Mag. 6·0.	Mag. 7·5.	Mag. 8·5.	Mag. 9·0.	Mag. 9·4.	Bright.	Gradients (See § 24). Mean.	Faint Stars.
h. h.								
0− 3	−6·3	−5·5	−2·1	−2·6	+0·9	−0·5	−2·07	−3·3
3− 6	+0·7	+0·4	+1·7	+0·4	+0·1	+0·2	−0·09	+1·8
6− 9	+1·4	−2·5	+0·9	+0·9	+1·9	+2·6	−0·81	−1·1
9−12	−2·1	+1·6	−0·4	+0·6	−0·1	−2·5	−0·09	−0·3
12−15	−1·1	−2·2	−0·4	−1·0	+1·3	+0·7	−0·72	−1·9
15−18	+1·5	+0·6	+0·5	+1·5	−1·9	+0·6	+0·45	+2·7
18−21	−0·9	−0·8	−1·4	+0·7	−0·3	−0·1	−0·23	−1·2
21−24	−2·9	−0·8	+0·7	−1·0	+0·3	−1·4	−0·81	+0·4
Mean	−1·21	−1·15	−0·06	−0·06	+0·28	−0·05	−0·55	−0·36

TAB

Mean Results

(The unit is

Octant of R.A.	Interval in Years.	6·0. E.	6·0. L.	6·0. L−E.	7·5. E.	7·5. L.	7·5. L−E.
h. h.							
0− 3	5·2	+74·7	+42·2	−32·5	+66·3	+37·6	−28·7
3− 6	5·2	+36·1	+39·9	+ 3·8	+18·9	+21·0	+ 2·1
6− 9	4·9	+42·4	+49·1	+ 6·7	+38·7	+26·6	−12·1
9−12	5·3	+43·2	+32·1	−11·1	+22·8	+31·2	+ 8·4
12−15	4·5	+36·6	+31·6	− 5·0	+35·8	+25·7	−10·1
15−18	4·6	+44·5	+51·3	+ 6·8	+26·2	+28·9	+ 2·7
18−21	4·1	+47·0	+41·5	− 5·5	+35·5	+32·2	− 3·3
21−24	4·0	+57·8	+46·3	−11·5	+39·8	+36·8	− 3·0

12. How far the mean quantities are accordant with the formula

$$0·55 \times (\text{mag.} - 9·0)$$

may be seen from the following small table.

TABLE VIII.

Mag.	Obs.	Calc.	O.−C.
6·0	−1·21	−1·65	+0·44
7·5	−1·15	−0·83	−0·32
8·5	−0·06	−0·28	+0·22
9·0	−0·06	0·00	−0·06
9·4	+0·28	+0·22	+0·06

But it seems difficult to regard the quantities O.—C. as accidental. The change is abrupt at about magnitude 8·0 rather than gradual.

13. We shall presently return to the question whether the change is abrupt or gradual; meanwhile we may note that the more careful discussion of the observations has reduced the magnitude of the relative P.M. from the value 1·00 found by the rough computation of § 9 (*c*) to the value 0·55 per magnitude per year: or in seconds of time $-0^{s}·0013$ per magnitude per year, which is almost precisely the quantity found by Sir D. Gill (see § 4), *but of opposite sign.*

14. We will now turn to an independent piece of evidence which presented itself in the course of the work. Having occasion to refer to my earlier paper on the Cambridge magnitude equation (*Monthly Notices*, lx. p. 3), my attention was arrested by the paragraphs 11 and 13 concerning the variation of the magnitude equation with R.A. Paragraph 11 runs as follows :—

LE VI.

for all Zones.

0″·03 or 0s·00225.)

8·5.			9·0.			9·4.		
E.	L.	L−E.	E.	L.	L−E.	E.	L.	L−E.
+26·0	+14·9	−11·1	−2·7	−16·1	−13·4	−31·3	−26·4	+4·9
+10·3	+19·0	+ 8·7	−5·4	− 3·3	+ 2·1	−33·3	−33·0	+0·3
+14·6	+18·8	+ 4·2	−2·4	+ 2·0	+ 4·4	−34·6	−25·2	+9·4
+17·5	+15·2	− 2·3	−9·8	− 6·4	+ 3·4	−24·6	−25·3	−0·7
+16·2	+14·5	− 1·7	−2·3	− 6·7	− 4·4	−42·0	−36·3	+5·7
+10·6	+12·7	+ 2·1	−4·1	+ 2·9	+ 7·0	−23·8	−32·3	−8·5
+21·7	+15·3	− 6·4	+0·9	+ 4·0	+ 3·1	−25·8	−27·1	−1·3
+24·3	+26·9	+ 2·6	+4·0	+ 0·1	− 3·9	−29·5	−28·3	+1·2

"Returning now to the Cambridge personality one important question is, Does it vary with the R.A.? The answer seems to be in the affirmative; but the variation is small compared with the whole amount. It is also rather exceptional in character; and as yet I can suggest no physical reason for it. The personality is rather greater for 21^h-3^h R.A. and for 12^h-15^h R.A. than for the rest of the circuit, and is greater proportionally—*i.e.* the increase is greatest for bright stars."

[In § 12 of this paper there is an error in the last line but one: *for* $+0^s{\cdot}013$ *read* $-0^s{\cdot}013$. The correct sign was used, but the wrong one printed.]

In § 13 of the same paper the actual figures are given as follows:—

Mag.	0^h-3^h.	3^h-6^h.	6^h-9^h.	9^h-12^h.	12^h-15^h.	15^h-18^h.	18^h-21^h.	21^h-0^h.
	s	s	s	s	s	s	s	s
5·0 to 6·9	+·030	−·005	−·015	−·013	+·010	−·005	−·024	+·011
7·0 „ 7·9	+·021	−·014	+·004	−·006	+·008	−·019	−·006	+·004
8·0 „ 8·4	+·010	−·003	·000	+·001				
8·5 „ 8·9	+·006	+·004	·000	−·004				
9·0 „ 9·4	−·010	+·006	−·002	+·007				

15. Now it occurred to me that perhaps the explanation of these differences lay in a difference of date at which the *Cambridge* observations were made. In meridian observing it is a common experience to find certain parts of the catalogue "cleared off" before others; and any systematic proper motions might thus introduce apparent systematic errors. The dates of the Cambridge observations were examined in two ways:

(A) The mean date was formed for each catalogue page (of fifty stars of all magnitudes) which contained the beginning of a new hour, *i.e.* the pages containing $0^h\,0^m$, $1^h\,0^m$, $2^h\,0^m$, and so on. The mean of the *four* pages, $0^h\,0^m$, $1^h\,0^m$, $2^h\,0^m$, $3^h\,0^m$, was

considered to represent the approximate mean date of the observations in the octant $0^h - 3^h$; and so for other octants.

(B) The epochs of observation for all the bright stars (5·0 to 8·0) in each octant were set down and the mean taken.

The results are given in Table IX., wherein is also given the mean date of the Oxford plates used for the paper above quoted. It will be seen that the variations of date are chiefly in the Cambridge observations, and are small. As the discrepancy under consideration is greatest for bright stars, it seems preferable to adopt column (B) in forming the interval between the Cambridge observations and the Oxford plates during which the effects of P.M. will accumulate.

TABLE IX.

Dates of the Observations used in the Paper of 1899 *November.*

	Cambridge (A)	Cambridge (B)	Oxford	Oxford −(B)	Diff. from Mean
h h				y	
0– 3	1880·8	1880·5	1896·3	15·8	−1·3
3– 6	80·5	79·5	96·3	16·8	−0·3
6– 9	80·0	78·2	95·8	17·6	+0·5
9–12	78·5	78·4	96·3	17·9	+0·8
12–15	80·8	79·4	95·6	16·2	−0·9
15–18	80·2	79·5	96·3	16·8	−0·3
18–21	80·1	77·3	95·7	18·4	+1·3
21–24	81·1	78·0	95·2	17·2	+0·1
Mean	...	1878·8	1895·8	17·1	...

16. Now if we arrange the quantities quoted in § 14 according to the value of the interval we get the following result:—

TABLE X.

Interval (Diff. from Mean)	Octant	Observed Mag. 5·0 to 6·9	Observed Mag. 7·0 to 7·9	Calculated Mag. 5·0 to 6·9	Calculated Mag. 7·0 to 7·9
	h h	s	s	s	s
−1·3	0– 3	+·030	+·021	+·005	+·003
−0·9	12–15	+·010	+·008	+·003	+·002
−0·3	3– 6	−·005	−·014	+·001	+·001
−0·3	15–18	−·005	−·019	+·001	+·001
+0·1	21–24	+·011	+·004	·000	·000
+0·5	6– 9	−·015	+·004	−·002	−·001
+0·8	9–12	−·013	−·006	−·003	−·002
+1·3	18–21	−·024	−·006	−·005	−·003

17. There certainly seems to be some ground for the explanation of these quantities, in part at any rate, by the relative

proper motions found above. Under the heading "calculated" are given the quantities deducible on the assumption that the relative P.M. is

$$0^{s}\cdot00124 \text{ per magnitude per year.}$$

If we add together the results for which the calculated effect is positive, and those for which it is negative, we get

	Mags. 5·0 to 6·9		Mags. 7·0 to 7·9	
Observed . .	+·030	−·052	−·004	−·008
Calculated .	+·010	−·010	+·007	−·006

The total sum of positive groups is $+^{s}\cdot026$ observed and $+^{s}\cdot017$ calculated; of negative groups $-^{s}\cdot016$ observed and $-^{s}\cdot060$ calculated. The accordance is thus far from perfect.

18. A better accordance is obtained if instead of assuming the same relative P.M. throughout we take separate results for each octant. But it is unprofitable to pursue this line of inquiry further at this point, for there is a disturbing cause affecting these figures. The clock-star list in use at Cambridge was changed on 1883 January 1; and Mr. Hinks has shown (in *Monthly Notices*, lvii. p. 474) that there was a resulting apparent increase of the Cambridge R.A.'s by $+0^{s}\cdot030$.

But his discussion takes no account of the present phenomenon, and may require modification in the light of it. The inquiry should be reopened and will need an elaborate piece of work which cannot be undertaken just at present. We must be content with the qualified confirmation of our results indicated in Table X.

19. Returning then to the main conclusion of § 13—

In declination + 25° *to* + 30° *the R.A.'s of the brighter stars are decreasing relatively to the fainter (between the limits of magnitude* 5·0 *to* 9·5) *at the rate of* $0^{s}\cdot00124$ *per magnitude per year*—

and comparing this with the conclusion of Sir D. Gill, that

In declinations −40° *to* −52° *the R.A.'s of the brighter stars are increasing relatively to the fainter (between the limits of magnitude* 4·0 *and* 8·0) *at the rate of* $0^{s}\cdot0014$ *per magnitude per year*,

the question arises, whether these two facts concerning particular belts of the heavens give us any information as to the general nature of the phenomenon. It seems to me that the difference of sign proves conclusively that we are *not* in the presence of what Sir D. Gill calls "an apparent rotation of the brighter fixed stars as a whole with respect to fainter stars as a whole." If there is such a rotation about any axis whatever we may resolve the minute effects observed hitherto into effects due to the rotations about two axes, one parallel to the axis of the Earth and the other perpendicular to it. The latter will

produce zero effect in R.A. for the mean of a zone of stars of the same declination (assuming the system of equal weighting adopted by Sir D. Gill and also in the present paper); while the former would produce a rotational effect which should have *the same sign for all zones.* The direct contradiction of sign in the northern and southern hemispheres seems to put this idea of a general rotation out of court.

20. Before proceeding to make any suggestion in place of it, let us consider again what is involved in the system of equal weighting in R.A. adopted by Sir D. Gill. He adopted it to eliminate the effect of the Sun's motion through space. I may perhaps quote a few words from a letter of his to me (dated 1902 Sept. 10) where he puts his point clearly.

"The parallactic motion due to the Sun's motion through space is :

$$\mu_\alpha = + \mathrm{V}\varpi \cos \mathrm{D} \sec \delta \sin (\alpha - \mathrm{A})$$
$$\mu_\delta = - \mathrm{V}\varpi \{\cos \delta \sin \mathrm{D} - \sin \delta \cos \mathrm{D} \cos (\alpha - \mathrm{A})\}$$

where V is the Sun's velocity expressed in radii of the Earth's orbit, A and D the R.A. and Decl. of the point in the heavens towards which the Sun is moving, and ϖ the annual parallax of a star whose R.A. and Decl. are α and δ.

"Now μ_α disappears in the mean proper motion of stars of the same Decl. Symmetrically distributed in R.A. μ_δ does not; hence the comparative simplicity of discussing R.A. only. Of course the complete discussion must involve the discussion of the declinations, but the result can only be derived by successive approximations."

21. Now, it is only true that "μ_α disappears in the mean proper motion of stars of the same Decl. symmetrically distributed in R.A." *if ϖ remains constant*; but if ϖ varies the mean will not generally be zero. Sir David Gill has selected stars of constant *magnitude*, but does it follow that they have a constant *parallax*? We have very little information on the point, and what we have is rather against this conclusion than for it. Not to speak of individual or exceptional cases, we have learnt, from the work on the "Cape Photographic Durchmusterung," that there is a systematic difference between the visual and photographic magnitudes of stars, depending on their galactic latitude. Is it the visual or the photographic magnitude which indicates the parallax? It is true the difference is small compared with either; but this is only one piece of evidence of *structure* in the universe, and directly the idea of structure is introduced we must admit that the parallax of a star will depend not only on its magnitude but on its *position.* If ϖ varies with δ and α the mean value of μ_α is no longer zero on the system of equal weights.

22. It seems possible, then, that Sir David Gill has indicated, not a general rotation in the universe, but a method of gauging its detailed structure; a method of elementary sim-

plicity and of the gravest importance. We must set to work to determine the gradient of relative proper motions as we pass from bright to faint stars in every quarter of the heavens, and study the results as they accumulate. In particular it would seem almost imperative that, as soon as possible after the Astrographic Catalogue, or any portion of it, is completed, it should be recommenced with a view to the determination of these proper motions.

23. But although our present information is scanty, it is, perhaps, not too early to anticipate a possible reason for the difference of sign found in the N. and S. hemispheres. The apex of the Sun's motion is at 18^h R.A. The Milky Way rises highest into the northern hemisphere about 0^h, where the effect of the parallactic motion is in the direction of increasing R.A.; while stars about 12^h, with decreasing R.A., are at nearly the limit of distance from the galaxy. These conditions are, of course, inverted in the southern hemisphere.

24. Let us consider in detail whether this may afford the clue to the cause of the phenomena under consideration. Returning to Table VII. the last three columns give three "gradients" formed as follows: Subtracting from the mean of the results for magnitudes 6·0 and 7·5 the mean of the three results for 8·5, 9·0, and 9·4, and dividing the difference by 2·2 (the difference of the mean magnitudes) we get the mean relative P.M. for one magnitude, which is called the mean gradient. But as there seems some reason to suppose that the gradient is not uniform two other columns have been formed: that for "Bright" stars by comparing the results for 6·0 and 7·5 and dividing by 1·5; and that for "Faint" stars by comparing 8·5 and 9·4 and dividing by 0·9. The accidental errors of these columns are of course larger than that for the mean.

25. Now if we are to explain these relative P.M.'s in terms of the Sun's motion in space, we must consider the parallactic factors for the eight octants. In the expression given above

$$\mu_a = V\varpi \cos D \sec \delta \sin (a - A)$$

we assume

V cos D constant for the present

[It is of course possible that V cos D, which is the motion of the Sun *relative to the stars*, may vary for different regions of the sky; and in fact some general suppositions of this kind have come under consideration in the course of the work, but were discarded later.]

and sec δ is constant for our zone

Hence we need only consider the factor

$$\varpi \sin (a - A)$$

If we divide the "gradient" by sin $(a - A)$ we get quantities

proportional to ϖ, which now represents the *difference of parallax per magnitude.* This division is performed in Table XI., and the galactic latitude of the group is added for comparison.

TABLE XI.

Octant.	Sin $(a-A)$.	Differences of ϖ per mag. 6·0–7·5.	Mean.	8·0–9·4.	Galactic Latitude.
h. h.					°
0–3	+·9	−0·6	−2·3	−3·7	−34
3–6	+·4	+0·5	−0·2	+4·5	−13
6–9	−·4	−6·5	+2·0	+2·8	+22
9–12	−·9	+2·8	+0·1	+0·3	+61
12–15	−·9	−0·8	+0·8	+2·1	+80
15–18	−·4	−1·5	−1·1	−6·8	+40
18–21	+·4	−0·2	−0·6	−3·0	+ 2
21–24	+·9	−1·6	−0·9	+0·4	−25

26. Looking now first at the column "Mean" (the others being affected by larger accidental errors), let us arrange the results in order of galactic latitude (taking no account of sign in this latitude).

Gal. Lat.	+2°	−13°	+22°	−25°	−34°	+40°	+61°	+80°
Mean	−0·6	−0·2	+2·0	−0·9	−2·3	−1·1	+0·1	+0·8

But for the positive sign at +22° these would fall into fairly regular sequence, all the quantities near the galaxy being negative and those near its poles positive; unless the + sign at +22° is an indication (allowing for accidental errors) of a return to a positive sign close to the galaxy.

27. The physical meaning of these figures would be, that within 24° of the Milky Way, and also near its poles, a bright star is nearer us on the average than a faint; but that for intermediate galactic latitudes (25°–50° say), and for a certain range of magnitudes, the *faint stars are actually nearer us than the bright.* And we have now to consider how this could be explained.

28. First consider a number of stars of the same intrinsic brightness (for brevity say the same *size*) scattered uniformly through space. When regarded from any view-point the apparent brightness of any of them varies as $1/(\text{dist.})^2$; or if 2·512 be denoted by a^2, the magnitude by m, and the distance by d

$$a-2m = \text{brightness} = C^2d^{-2}$$
$$\text{or } d = Ca^m$$

where C is a constant.

The total number of stars within a distance d varies as d^3, or

a^{3m}, and the number between magnitudes m and $m+1$ thus varies as $a^{3m+3}-a^{3m}$), which itself varies as a^{3m}. Hence we have for stars of magnitude m

$$\text{Parallax} = Aa^{-m}$$
$$\text{Number} = Ba^{3m}$$

where A and B are constants depending on the number of stars per unit volume and the intrinsic brightness of each.

29. Secondly, suppose we have two such systems superposed, the constants for one being A_1 B_1, for the other A_2 and B_2. Then we have for stars of magnitude m

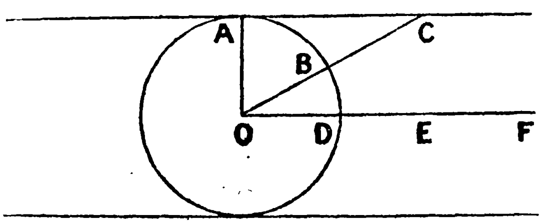
$$B_1a^{3m} \text{ stars of parallax } A_1a^{-m}$$
$$B_2a^{3m} \quad \text{,,} \qquad \text{,,} \qquad A_2a^{-m}$$

So that the *mean* parallax for magnitude m is now

$$\frac{B_1A_1+B_2A_2}{B_1+B_2}a^{-m} = A_3a^{-m}$$

i.e. it follows the same law of decrease with magnitude as before only with a different constant A_3 instead of A_1 or A_2, and intermediate between them, since B_1 and B_2 are both positive.

30. Now imagine two systems of stars to be superposed, one a set of small (*i.e.* intrinsically faint) stars uniformly scattered round the Sun in all directions as represented by the circle ABD, the Sun being at O.



Let the central plane of the galaxy be in the direction ODEF, and suppose the galaxy to be a superposed system of large stars (*i.e.* intrinsically brighter than those of the "solar cluster," as we may call the first set perhaps).

Consider the relation between brightness and parallax along three radii, OA, OBC, ODE.

Along OA we have both systems freely mixed, and the relation between magnitude and parallax will follow the usual law.

$$\text{Mean parallax} = A_3a^{-m}$$

where A_3 is, as shown above, intermediate between A_1 and A_2 $(A_1 > A_2)$.

Along OBC, however, the stars are only mixed as far as B; and from B to C we have some *large stars* which are thus

reckoned as near us. In fact there is a discontinuity in the law connecting magnitude and parallax. We start with a series

$$A_3 \qquad A_3 a^{-1} \qquad A_3 a^{-2} \qquad A_3 a^{-3} \ . \ . \ . \ A_3 a^{-r}$$

and we must ultimately change to the series

$$A_2 a^{-(r+1)}, \qquad A_2 a^{-(r+2)}, \ \&c.$$

where A_2 is less than A_3.

So that though the early terms of the series decrease, and the later terms will ultimately decrease, we have near a certain magnitude r an *increase*, viz.

$$A_2 a^{-(r+1)} > A_3 a^{-r}$$

which will be the case provided

$$A_2 > a A_3, \text{ i.e.} > 2{\cdot}512 \ A_3.$$

An abrupt change at a definite magnitude is of course only taken for illustration. The discontinuity will no doubt be spread over several magnitudes; but it is clear that we may in this way get, for a limited range of magnitudes, an *increase* of parallax with decreasing brightness instead of the usual *decrease*.

31. Finally consider the radius ODEF. To make the explanation accord with what has been assumed above, we must show cause why there should be a return to the normal state of things in directions near the galaxy itself. Now the existence of the anomaly depends upon the inequality

$$A_2 > a A_3$$

and from above we have

$$A_3 = \frac{B_1 A_1 + B_2 A_2}{B_1 + B_2}$$

B_1 and B_2 being proportional to the numbers of stars of each class. As we approach the galaxy we include more stars of the galactic type, and hence we may assume that B_2 increases; so that A_3 approaches nearer A_2, and the inequality no longer holds.

32. This, then, is the rough suggestion for the phenomena observed. It seems worth putting forward as a "working hypothesis," which can be at once tested in several ways, and especially the proper motion in *declination* must be examined for the same plates as have been examined above in R.A.

But this must be deferred to a future paper. Meanwhile the results obtained are so interesting that it seems desirable to publish them at once; and I put this hypothesis along with them, not as fully established, but as one with which it is desirable to compare future results, until it is definitely contradicted by any of them. The former hypothesis of Sir David Gill, of a general rotation of the bright stars with respect to faint stars, turns out

(if the results above given may be accepted) not to be well founded; but I would be among the first to be grateful to him for enunciating it, since it was the means of drawing my attention at once to a line of work which seems likely to bear fruitful results.

And I will venture to add here another word or two of a personal kind. In No. 517 of the *Astronomical Journal* Professor Lewis Boss called attention to the fact that in determining the Cambridge magnitude equation I had treated proper motions as accidental errors, which would disappear in the mean of a number of stars. "Unless this statement is surrounded by several qualifications," wrote Professor Boss, "it is by no means correct," and he went on to specify the qualifications, among which considerations of the Sun's parallactic motion were prominent. Some misunderstandings of what had actually been done in the Oxford work, and, on my part, of what Professor Boss really meant, gave rise to a discussion which turned out to be irrelevant to the real issue (*Astronomical Journal*, Nos. 519, 521, 523). It has been, I hope, satisfactorily closed. But now that I properly understand the real meaning of Professor Boss's remarks with regard to the parallactic motion, it gives me peculiar pleasure to be among the first to confirm their soundness and value in a new way.

A Note relating to the Preservation of Negatives.
By F. A. Bellamy.

Some weeks ago I had occasion to examine a negative of a comet taken in 1899, and upon withdrawing it from its envelope, similar to those used at the University Observatory, Oxford, for storing plates taken for the Astrographic Catalogue, it was immediately noticed by Mr. H. C. Plummer and myself that a representation or impression of the description of the negative written outside the thick envelope had been conveyed to the gelatine film, and almost every letter and figure—though blurred in character, much as a heavily written word would appear on blotting-paper immediately used—could be read *from the plate.* It should be noted that no mark is visible on the *inside* of the envelope to show that the ink had soaked through.

I have since examined a large number of our plates, and I have, I am pleased to state, found very few cases; I may say in general terms that no plate which apparently had a short development and those taken on dark nights—in fact, those that had a bright, clear appearance, and upon which the stars and réseau lines stood in relief; as in the carbon process—showed any sign of this transmitted writing; but the plates that did show it